Synchronization of Fireflies: Exploring the Role of Mathematics in Collective Behaviors

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Abstract

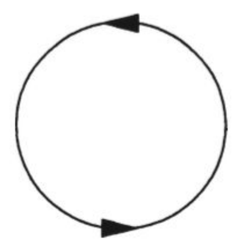
Fireflies are known for their abilities to emit bioluminescent flashes. They can synchronize their flashing patterns when a group of fireflies are in close proximity.

Through this poster, we aim to showcase the role of mathematics in understanding the synchronization of fireflies. By using a mathematical models, people can gain a deeper understanding the mechanisms and apply it to other areas of research.

Background

Differential equations: Equation involving derivatives

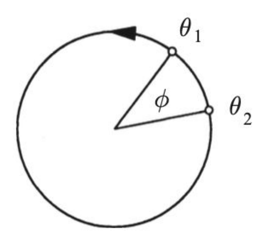
Notation: 1) dθ/dt = $\dot{\theta} = f(\theta)$,
2) x* means the fixed points of f(θ).



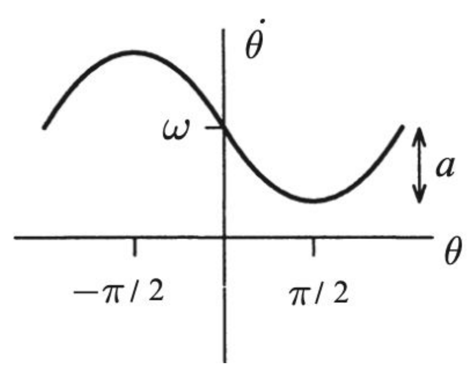
Vector field on the circle :
- Unique velocity vector to each point on the circle.
- Periodicity (The solution is periodic.)

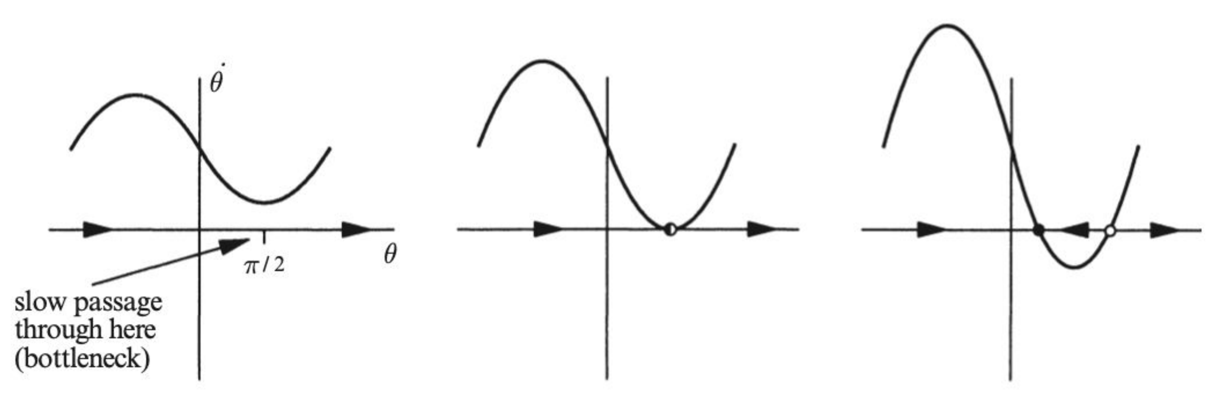
Phase angle: describes the position of the oscillator within its cycle
Phase difference: difference between phase angles

Nonuniform Oscillator:

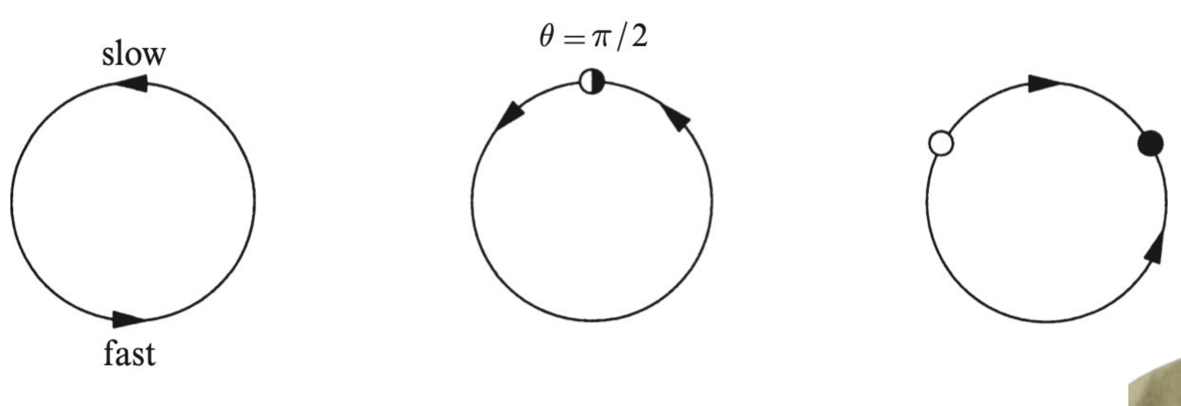


$$\dot{\theta} = \omega - \overline{a \sin\theta}$$





(a) $a < \omega$ (b) $a = \omega$ (c) $a > \omega$



(a) $a < \omega$ (b) $a = \omega$ (c) $a > \omega$

The Main Problem/Goal

How can we use a simple model to model the periodic and collective flashing behavior of fireflies?

Simple Ermentrout and Rinzel model(1984)

A model of one firefly's flashing model rhythm and its response to stimuli.

$$\boxed{\dot{\theta} = \omega + A\sin(\Theta - \theta)} \qquad \dot{\Theta} = \Omega$$

θ(t) is the **phase angle** of the firefly's flashing rhythm.
Θ(t) is the phase angle of another firefly's flashing rhythm.
θ = 0 when a flash is emitted by the firefly.
Θ = 0 when a flash is emitted by the stimulus.
w = constant natural frequency

If the **stimulus** Θ (another firefly) is ahead in the cycle, then we assume that the firefly speeds up to catch up, in an attempt to synchronize. And conversely, slow down when the stimulus is behind.

Resetting strength: A = ability to modify its instantaneous frequency.

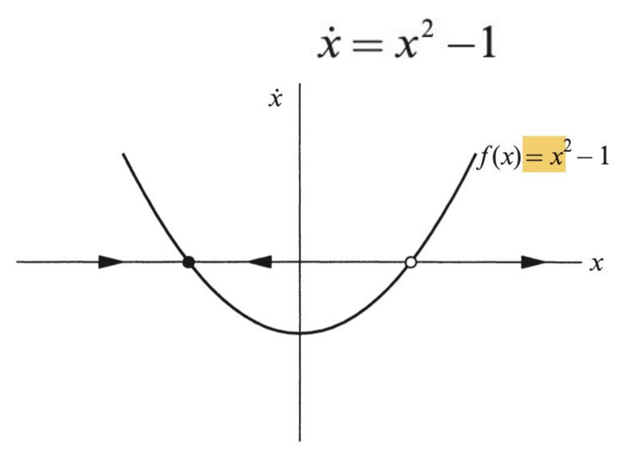
ϕ = Θ - θ is the **phase difference.**

If stimulus Θ is ahead of θ, the fireflies *speed up* $\boxed{\dot{\theta} > \omega}$
0 < Θ - θ < π
If stimulus Θ is behind of θ, the fireflies *slow dow* $\boxed{\dot{\theta} < w}$
π < Θ - θ > 2π

More definition

Coupling strength : The coupling strength refers to the intensity or magnitude of the interaction between two or more oscillator

Fixed point: Equilibrium solution/ At a fixed point, the derivatives of the state variables are zero, indicating that the system is at rest.



Stable fixed point: small disturbances away from t damp out in time.
Unstable fixed point: disturbances grow in time.

When does synchronization occur?

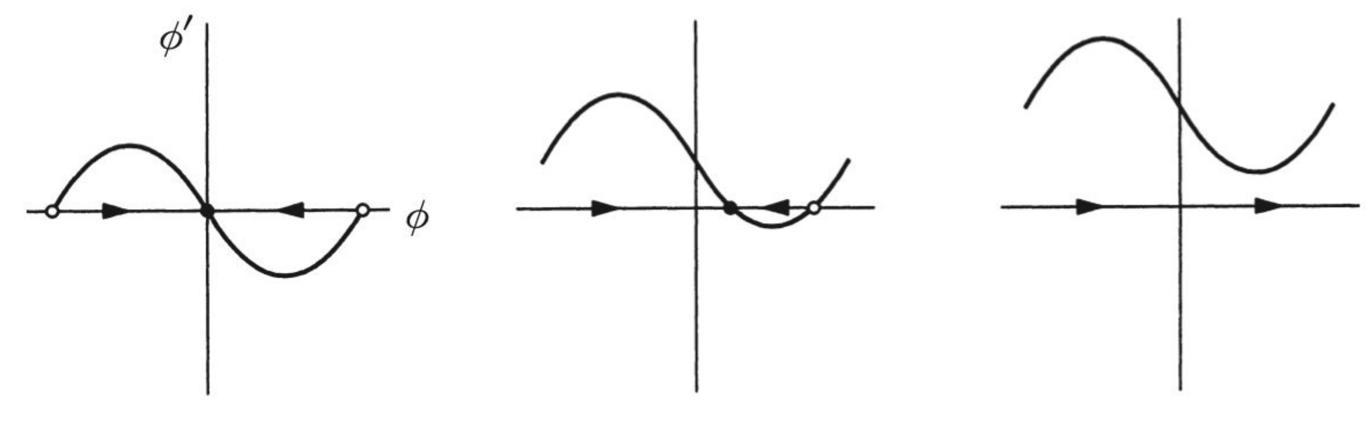
ϕ = Θ - θ
When **ϕ = 0:** Both fireflies synchronized with zero phase difference = simultaneously flash

$$\dot{\phi} = \dot{\Theta} - \dot{\theta} = \Omega - \omega - A\sin\phi,$$

$$\phi' = \mu - \sin\phi$$

ϕ' is the derivative of the phase angle with respect to time

μ is coupling strength



(a) $\mu = 0$ (b) $0 < \mu < 1$ (c) $\mu > 1$

When a), all trajectory flow toward a **stable fixed point** ϕ=0. They flash simultaneously.
When b), all trajectory has a **stable** and **unstable** fixed point. And the ϕ*>0, firefly's rhythm is a **phase locked.**
When c), a saddle-node bifurcation coalesce at μ=1. And all fixed points are gone, which means ϕ will increase and decrease over time. The collective behavior of the fireflies may become unstable.

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References

Strogatz, S. H. (n.d.). Flows on the circle. In *Nonlinear Dynamics and Chaos: With applications to physics, biology, chemistry, and engineering*. essay.